1. Stress Tunneling Coefficient Beyond the Kane Kink

One expects the onset of direct tunneling to the (000) conduction band to occur at a reverse bias voltage

$$V_k = -[E_g(000) - E_g(111) - \zeta_n]/e. \quad (5)$$

Since the pressure coefficient for the direct gap $E_g(000)$ is larger than that for the indirect gap $E_g(111)$,[8] that part of the uniaxial compression X which corresponds to a hydrostatic pressure $p = X/3$ causes the onset voltage V_k to increase. This results in an anomalously large negative value of $\Delta I/I$. As one goes to higher reverse bias, this contribution becomes relatively unimportant and the stress coefficient approaches the value determined by the stress-induced changes of $E_g(000)$ and the combined electron and hole effective mass $m^*(000)$. Figure 1 shows that these changes are determined mainly by the hydrostatic pressure part of the stress. The shear part does not change $E_g(000)$. It deforms, however, the effective mass spheres, which gives rise to a small contribution which is different for the two diode orientations.

Beyond the Kane kink the tunnel current is the sum of the direct current I_d and the indirect current I_i (see Fig. 4). Following Kane one has

$$I_d = C_d D(V - V_k) \exp(-\alpha), \quad (6)$$

with

$$\alpha = \lambda_d E_g^{3/2}(000) m^*(000)^{1/2}/F, \quad (7)$$

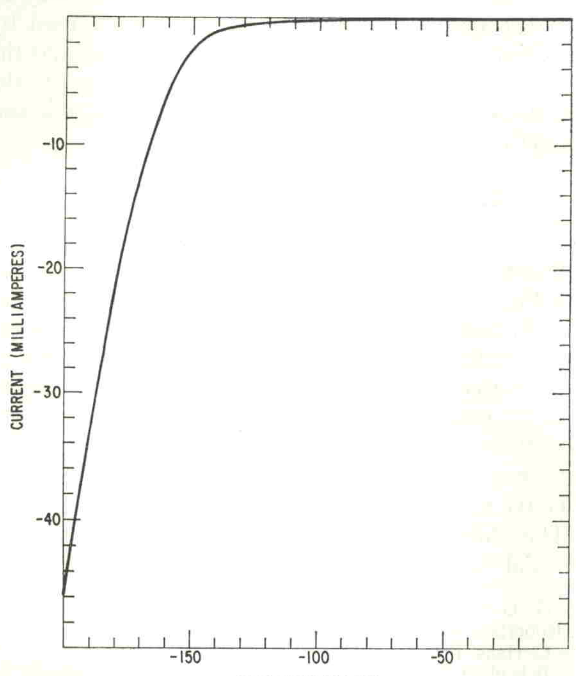

FIG. 3. Current as a function of reverse bias of one diode of sample 2 at 4.2°K. Note the sharp increase in current near $V = -140$ mV.

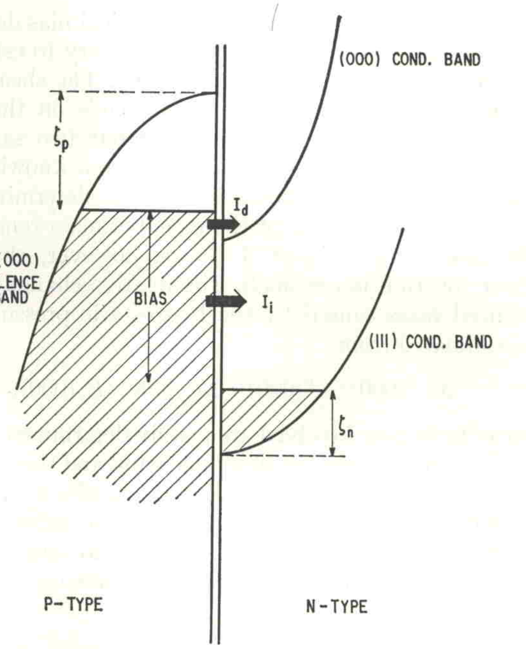

FIG. 4. Relative position of the bands and Fermi levels in a germanium tunnel diode for a reverse bias beyond the Kane kink.

and

$$D(V - V_k) = e(V - V_k) + \frac{E_g(000)}{4\alpha}\left\{1 - \exp\left[-\frac{4\alpha e}{E_g(000)}(V - V_k)\right]\right\}. \quad (8)$$

The relative change of the tunnel current is

$$\Delta I/I = (\Delta I_d + \Delta I_i)/(I_d + I_i). \quad (9)$$

It can be seen from Fig. 4 that beyond the Kane kink the indirect current is an appreciable fraction of the total current only over a small bias range. For the purpose of explaining the data we can, therefore, use for I_i and ΔI_i in Eq. (9) the extrapolations of the indirect current curves into the bias range beyond the Kane kink.

For the change of the direct current one obtains from Eqs. (6) and (7), neglecting the relatively minor change of C_d,

$$\Delta I_d = I_d \frac{1}{D}\frac{dD}{dV_k}\Delta V_k - I_d\alpha\left[\frac{3}{2}\frac{\Delta E_g(000)}{E_g(000)} + \frac{1}{2}\frac{\Delta m^*(000)}{m^*(000)} - \frac{\Delta F}{F}\right]. \quad (10)$$

The first term in Eq. (10) is responsible for the sharp maximum of the stress coefficient near V_k, and the second term determines its asymptotic value at large reverse bias. The quantity ΔV_k is

$$\Delta V_k = -(\Xi_{000} - \Xi_{111})X/3, \quad (11)$$

where $\Xi_{000} = 12 \times 10^{-12}$ Vcm²/dyn and $\Xi_{111} = 5 \times 10^{-12}$ Vcm²/dyn are the pressure coefficients for the direct and the indirect band gap, respectively.[8]

Before one can calculate the theoretical bias dependence of the stress coefficient, it is necessary to estimate α and the value of $\Delta m^*(000)/m^*(000)$. The shear contribution to the latter quantity depends on the field direction and hence is different for our two samples. Its magnitude cannot be obtained without knowing the various deformation potentials which determine the stress-induced changes of the effective mass tensors at the zone center. Figure 1 shows, however, that the shear contribution is small. The relative change of the reduced mass caused by the hydrostatic pressure can be estimated from

$$\Delta m^*(000)/m^*(000) = \Delta E_g(000)/E_g(000), \qquad (12)$$

since the masses involved are mainly determined by the interaction between the light hole band and the conduction band at the zone center.[14] The quantity α was then determined by fitting Eq. (10) and the experimental hydrostatic pressure curve of sample 1 at large reverse bias. From this fit, the value $\alpha = 17.6$ was obtained for $V = -300$ mV.

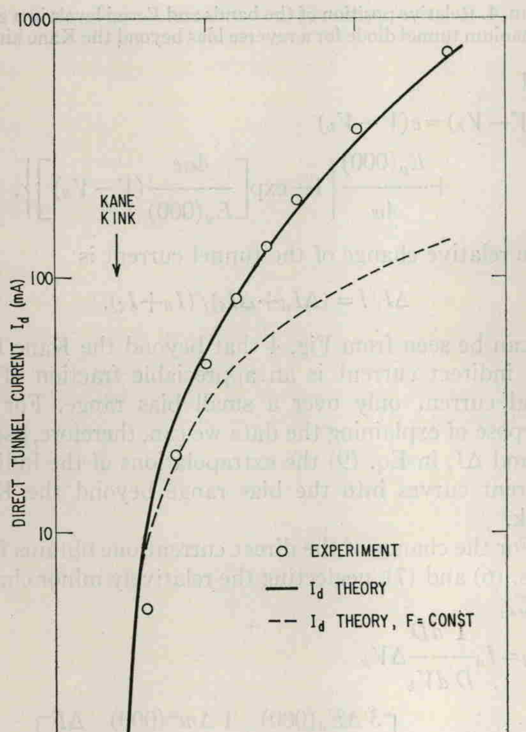

FIG. 5. Comparison between theory and experiment of the bias dependence of the direct tunnel current beyond the Kane kink. The factor C_d of Eq. (1) (see text) has been chosen to fit the absolute magnitude of the theoretical and experimental I_d near -220 mV reverse bias.

As an independent check, the bias dependence of the direct tunnel current itself was compared with Eqs. (6) and (7) using this α. The comparison with experiment is shown in Fig. 5. The constant factor C_d of Eq. (6) was chosen to fit the magnitudes of the measured and the calculated current. It is seen that this value of α predicts the shape of the I-V characteristic quite well. This agreement was not noted by Morgan and Kane[6] because they assumed the junction field F did not change with bias. Even though F varies by only about 10%, the value of the exponential factor varies by a factor of 6 over the bias range of interest.

The theoretical stress coefficient is compared with experiment in Fig. 6. It is seen that there is good qualitative agreement except that the theoretical maximum is sharper than the experimental curve. There are several effects which will cause a smearing out of the theoretical curve. (1) Thermal fluctuation will cause about a 1.5-mV broadening. (2) Random fluctuations of the impurity concentrations on a microscopic scale will cause local fluctuations in ζ_n. These will correspond to a range of V_k values rather than a unique value as was assumed. (3) There may also be a nonuniform built-in stress in the diodes. Since $E_g(000)$ and $E_g(111)$ depend differently on stress, this would also cause a spread in V_k.

For pure germanium $E_g(000) - E_g(111) = 144$ mV.[15] For our samples, V_k should, therefore, occur at 124 mV. The observed V_k is clearly larger than 136 mV. This discrepancy may be due to a depression of the (111) conduction band relative to the (000) conduction band due to the large impurity concentration,[16] or it may be due to a permanent strain at the junction caused by the difference in lattice constant of the n-type and the p-type regions, or possibly by the difference in the thermal expansion coefficient of the dot material and the germanium.[17]

2. Stress Coefficient in the Indirect Tunneling Range

Except for the narrow voltage region at zero bias (see Fig. 2) the tunneling is almost entirely indirect for $V_k < V$. In this bias range the stress coefficients of the two samples differ strongly. This difference is due to the fact that in sample 1 all four valleys are equivalent with the respect to the junction field direction, whereas in sample 2 the two pairs of valleys which are shifted with respect to one another by shear have different effective mass components in the field direction.

The difference between the hydrostatic and the uniaxial stress coefficient of sample 1 (see Fig. 1) is due

[14] E. O. Kane, J. Phys. Chem. Solids **7**, 249 (1957). For the relationship between effective masses and energy gap, see also H. Ehrenreich, J. Appl. Phys. **32**, 2155 (1961).

[15] G. G. Macfarlane, T. P. McLean, J. E. Quarrington, and V. Roberts, Proc. Phys. Soc. (London) **71**, 863 (1958).

[16] C. Haas, Phys. Rev. **125**, 1965 (1962).

[17] It is clear that stresses of sufficient magnitude to account for this discrepancy can easily be encountered unless special precautions are taken to avoid them. For example, S. Zwerdling, B. Lax, L. M. Roth, and K. J. Button [Phys. Rev. **114**, 80 (1959)] measured $E_g(000) - E_g(111) = 0.154$ eV in strained material.